3.1.3. Demagnetizing Energy

The demagnetizing energy can be obtained by solving the magnetostatic boundary value problem for the magnetic surface pole distribution on two surfaces separated a distance L as is indicated in Figure 3.2(c). The solution requires only a slight variation on a problem already solved by Kittel.[4] Only the result will be reported here. It is approximately given by

$$\mathcal{E}_d = 1.1 \frac{DM_s^2}{L} \sin^2\theta .$$

The approximation results from terminating an infinite series. Again L is the slab thickness, D is the domain dimension, and M_s is the saturation magnetization.

3.1.4. Total Energy

From the results of Sections 3.1.1, 3.1.2, and 3.1.3, the total thermodynamic energy for the ferromagnetic material behind the shock front can be explicitly written. The total energy is

$$\mathcal{E}(D,\theta) = -M_s H_e \cos\theta + be \sin^2\theta + 1.1 \frac{DM_s^2}{L} \sin^2\theta + \frac{8\sqrt{A|be|}}{D} \sin^2\theta . \quad (3.7)$$

Equilibrium thermodynamics predicts that the energy expression, $\mathcal{E}(D,\theta)$, will be a minimum with respect to a variation of the internal coordinates, D and θ. Consider the domain width parameter first. Minimizing with respect to D gives

$$\frac{\partial \mathcal{E}}{\partial D} = 1.1 \frac{M_s^2}{L} \sin^2\theta - \frac{8\sqrt{A|be|}}{D^2} \sin^2\theta = 0 .$$

This yields an expression for the domain width.

$$D = \left(\frac{8L\sqrt{A|be|}}{1.1\, M_s^2}\right)^{1/2}$$

This can be substituted into Equation (3.7) giving

$$\mathcal{E}(\theta) = -M_s H_e \cos\theta + be \sin^2\theta + 2\left(\frac{8.8M_s^2\sqrt{A|be|}}{L}\right)^{1/2} \sin^2\theta$$

or

$$\mathcal{E}(\theta) = -M_s H_e \cos\theta + be \sin^2\theta + \gamma|e|^{1/4} \sin^2\theta \qquad (3.8)$$

where

$$\gamma = 2\left[\frac{8.8M_s^2\sqrt{A|b|}}{L}\right]^{1/2} .$$

The last term in Equation (3.8) will be called the equilibrium exchange and demagnetizing energy.

At this point a discussion of the results obtained so far is warranted. An estimate of the exchange constant can be obtained from molecular field theory.[22] This is

$$A \simeq \frac{3kT_c}{za}$$

where k is Boltzmann's constant, T_c is the Curie or Néel temperature, z is the number of nearest neighbors, and a is the lattice constant. This gives $A = 3 \times 10^{-7}$ erg/cm in YIG. At a strain of $-.01$ in YIG which corresponds to a shock pressure of about 25 kilobars, the predicted domain width is approximately 20 μron. This is in agreement with other work.[11] The equilibrium exchange and demagnetizing energy in Equation (3.8) is observed to increase as the fourth root of the strain while the induced anisotropy